



Trinity College Dublin

Coláiste na Tríonóide, Baile Átha Cliath

The University of Dublin

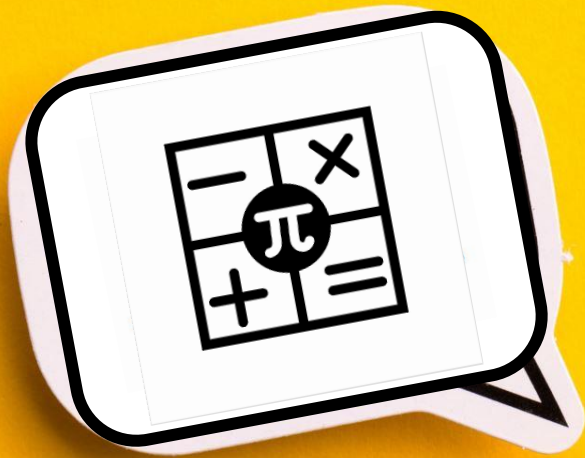
Mathematical Proficiency & Contextual Mathematics

InterEducation Workshop

Dr Aibhín Bray

28th November 2024

Who am I?



- Lecturer and researcher in the School of Education, Trinity College Dublin
- Trinity Access Programmes
- Maths Teacher



Learning Intentions

We are learning to...

- Understand the components of Mathematical Proficiency
- Identify and maximise opportunities for developing mathematical proficiency.
- Understand of rationale behind Realistic Maths Education and Bridge21
- Feel more confident structuring project-based maths activities



PLAN

Morning

- 10:00 **Set-up** : Ice breakers
- 10:10 **Warm-up**: Team formation
- 10:25 Mathematical Proficiency
- 11:25 Bridge21 and RME
- 11:45 Activity 1: **Investigate, Create,**
- 12:00 Activity 1: **Present**
- 12:10 Break

Afternoon

- 12:25 Activity 2
- 12:35 **Plan**
- 12:45 **Investigate, Create, iterate**
- 13:40 **Present** and
- 13:50 **Reflect**
- 14:00 End



Ice breaker

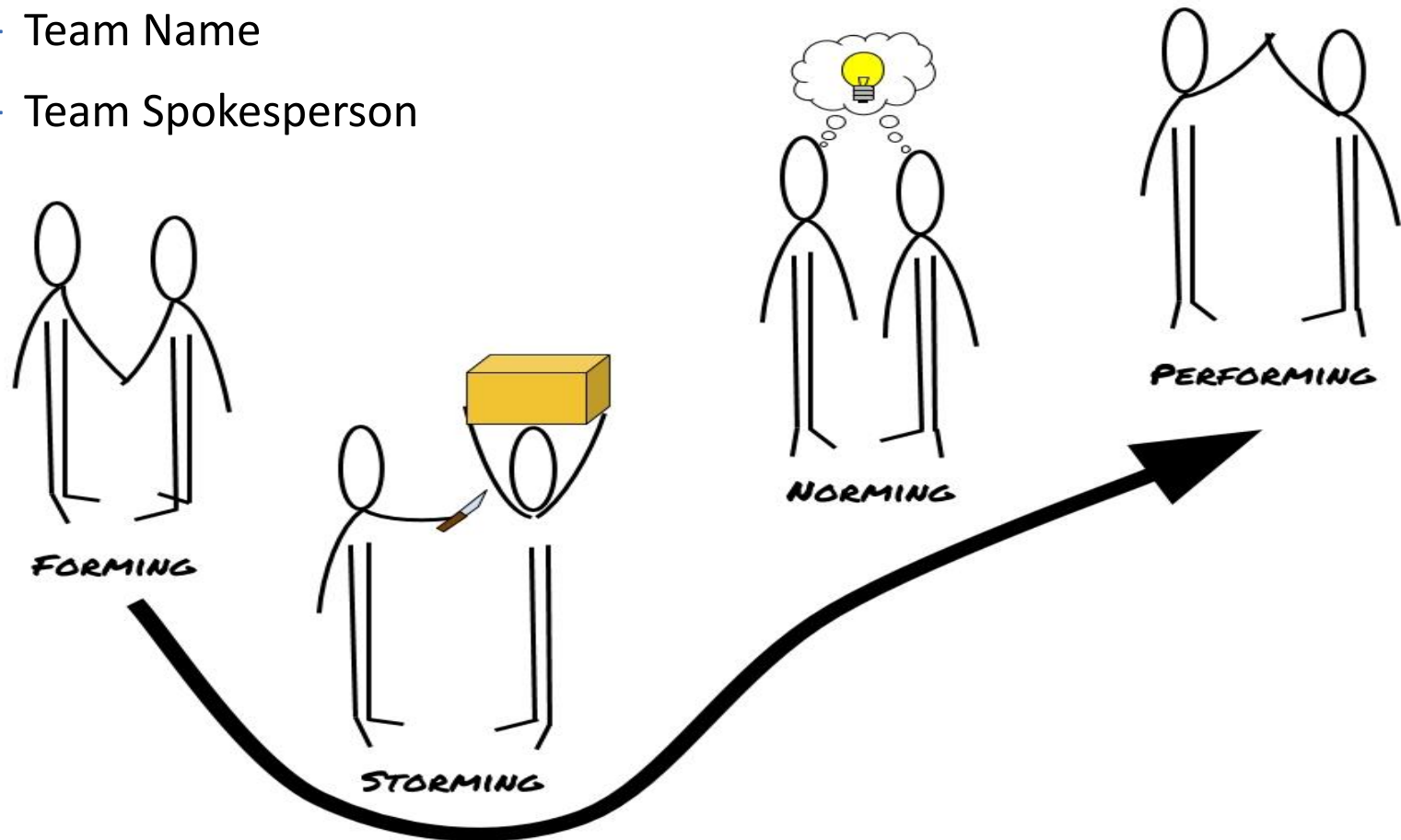
Knowing me, knowing you – 10 minutes



Team Formation

Group task – 2 minutes

- Team Name
- Team Spokesperson



Your warm-up task...

10 minutes

Problem:

How much would it cost to fill this room with €2 coins?





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Policy Background



What does it mean to be a proficient maths student

Do you agree?

Proficient

Proficient - Meaning:

Adjective. To be very skilled and capable at something.

Proficient students expect mathematics to make sense. They take an **active stance** in solving mathematical problems. When faced with a non-routine problem, they have the **courage** to plunge in and try something, and they have the **procedural and conceptual tools** to carry through. They are experimenters and inventors, and can **adapt** known strategies to new problems. They think strategically.

(Phil Daro, 2010, quoted in Burkhardt & Schoenfeld, 2019)

The Irish Curriculum – Lower Secondary

Junior Cycle Maths (2018)

The aim of junior cycle mathematics is to provide relevant and challenging opportunities for all students to become mathematically proficient:

1. conceptual understanding
2. procedural fluency
3. strategic competence
4. adaptive reasoning
5. productive disposition

(Kilpatrick et al., 2001, p117)



What do these mean?

Learning Intention:

To identify and maximise opportunities for developing mathematical proficiency.



Success Criteria:

I can explain the five components of mathematical proficiency.

I can identify which components of mathematical proficiency are developed by various tasks.

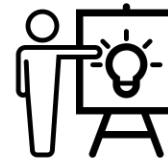
Activity

Engage with a task as if we were students in a lower second-level Maths class

10 mins



Examine the components of mathematical proficiency



Discuss how the components of mathematical proficiency can be developed using Tasks

10 mins



Discussion



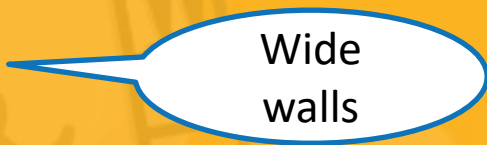
What features are important in **tasks/activities** for students to learn mathematics successfully?



5 mins

Properties of a “rich” task

(Ahmed, 1987)

- Must be accessible to everyone 
- Allows further challenges 
- Invites children to make decisions
- Involves children in speculating, hypothesis making, testing, proving, explaining, reflecting, interpreting
- Promotes discussion and communication
- Encourages originality and invention 
- Encourages ‘what if’ and ‘what if not’ questions
- Should be enjoyable
- Contains an element of surprise



Rich tasks

Rich vs. impoverished tasks (Malcolm Swan, Mathsfest 2012)

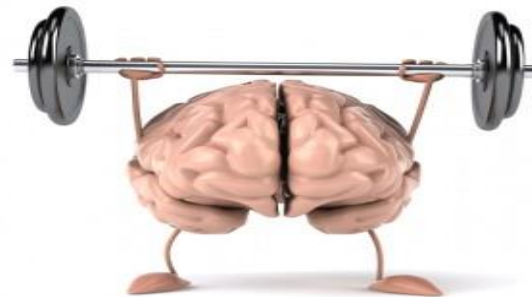
- Development of *fluency* can be embedded in **rich** tasks... but typical “define – show example – provide practice” sequences (in many textbooks) aiming at fluency are **impoverished** tasks.

And perhaps (John Mason, Mathsfest 2012)

- It is not the *task* that is rich ... but **whether it is used richly**
- What matters more than the particular answer is ...
 - **How** do you know?
 - **What** can you vary and still the same approach works?




Try this in pairs/groups

Someone has written a sentence containing only five words, the mean number of letters per word is 4, but none of the words has four letters. What might the sentence have been?



Properties of a “rich” task

(Ahmed, 1987)

- Must be accessible to everyone 
- Allows further challenges 
- Invites children to make decisions
- Involves children in speculating, hypothesis making, testing, proving, explaining, reflecting, interpreting
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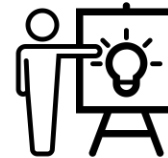
Activity

Engage with a task as if we were students in a lower second-level Maths class

10 mins



Examine the components of mathematical proficiency



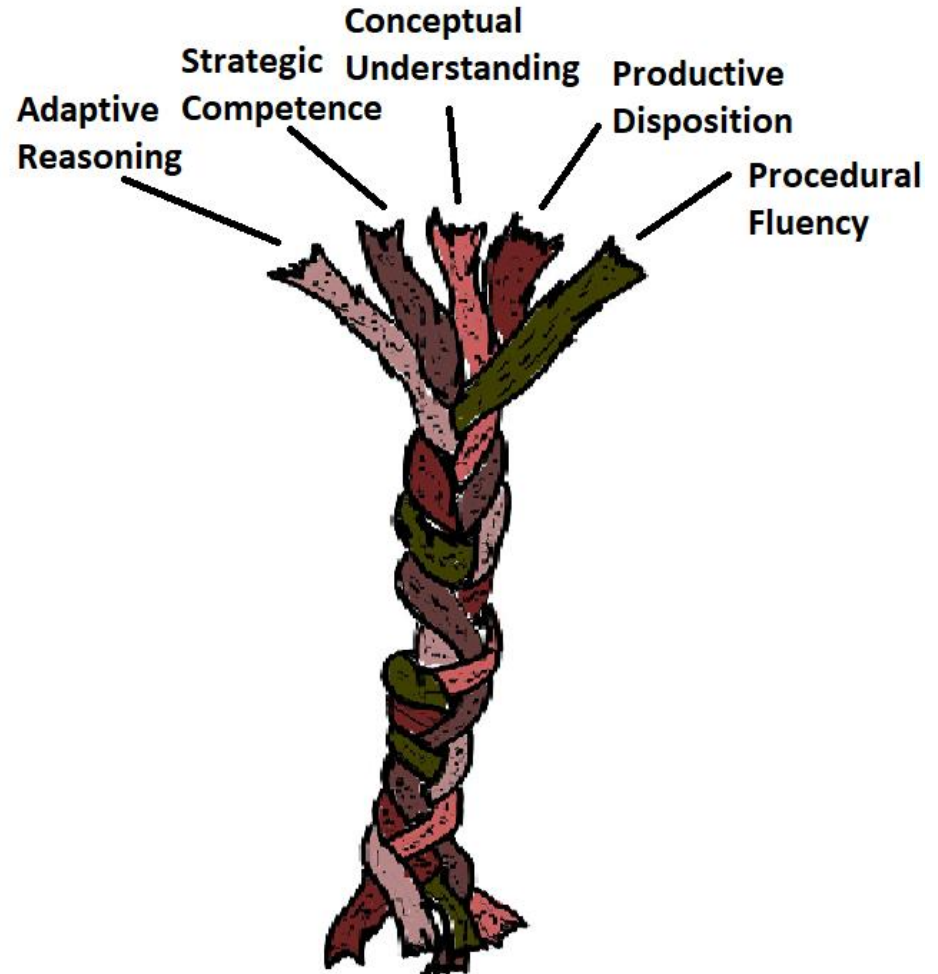
Discuss how the components of mathematical proficiency can be developed using Tasks

10 mins



Activity

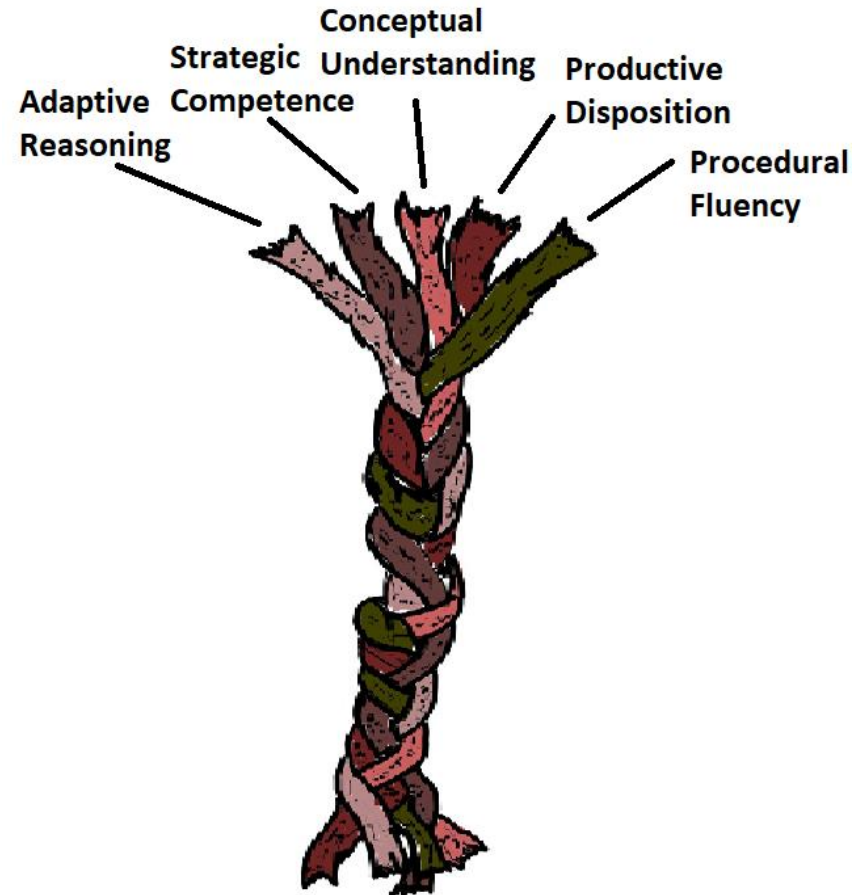
Examine the components of mathematical proficiency



Mathematical Proficiency

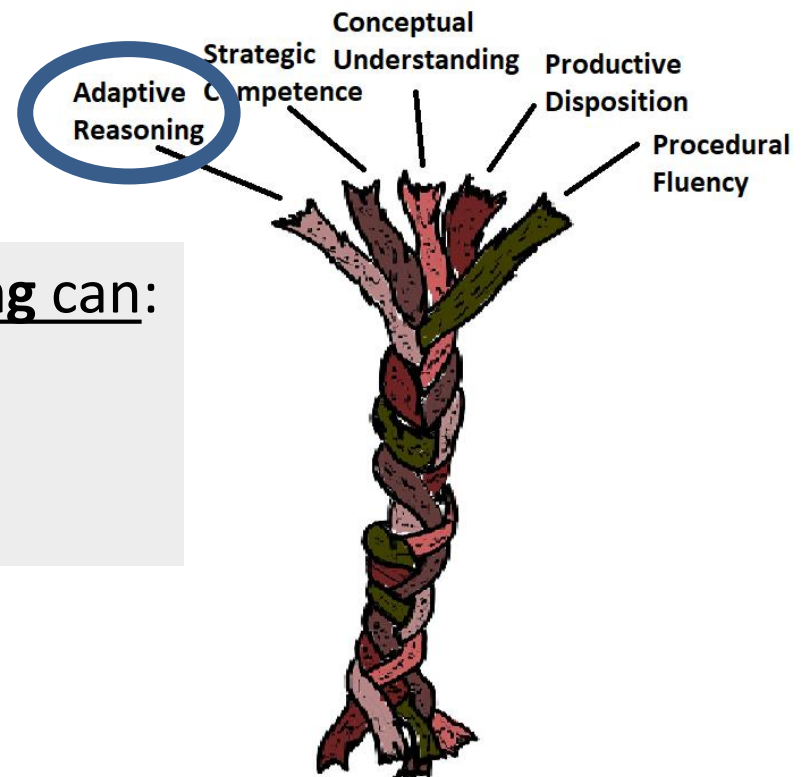
‘...mathematical proficiency is conceptualised not as a one-dimensional trait but as having *five interconnected and interwoven components.*’

Junior Cycle Mathematics Specification
p.5



Adaptive Reasoning

‘The capacity to think logically about the relationships among concepts and situations.’

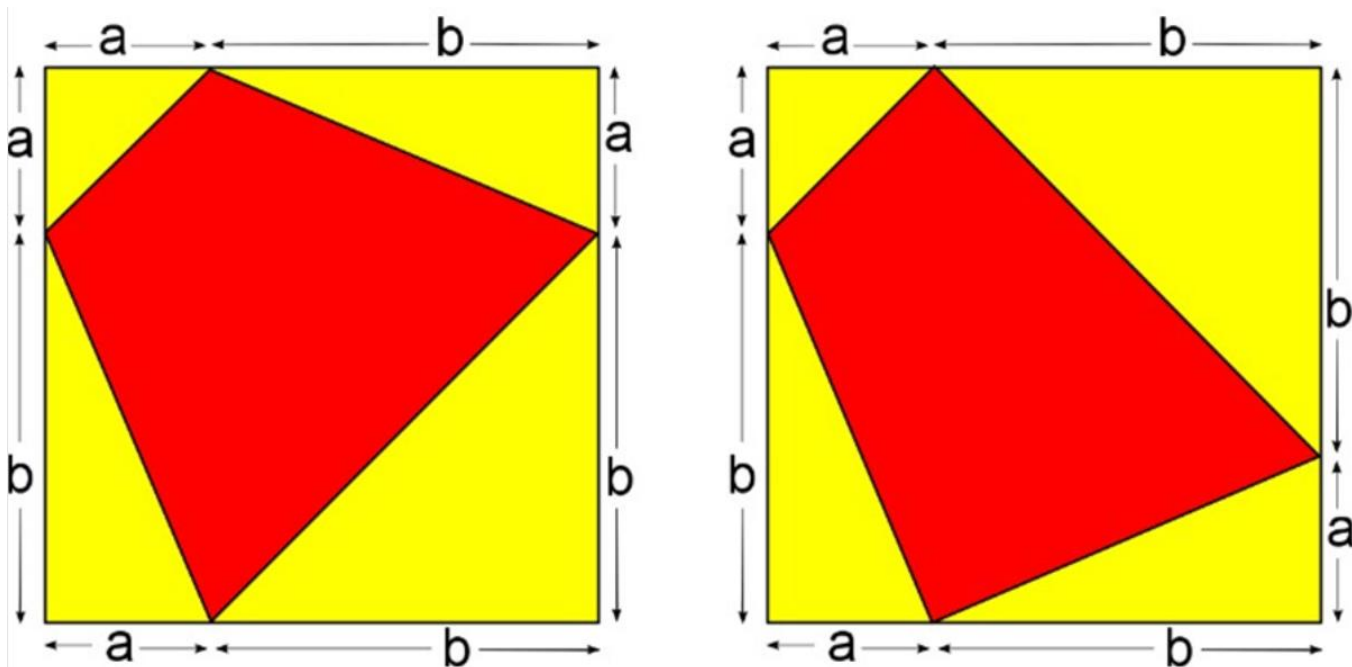


Students with **Adaptive Reasoning** can:

- Justify
- Explain their reasoning
- Prove

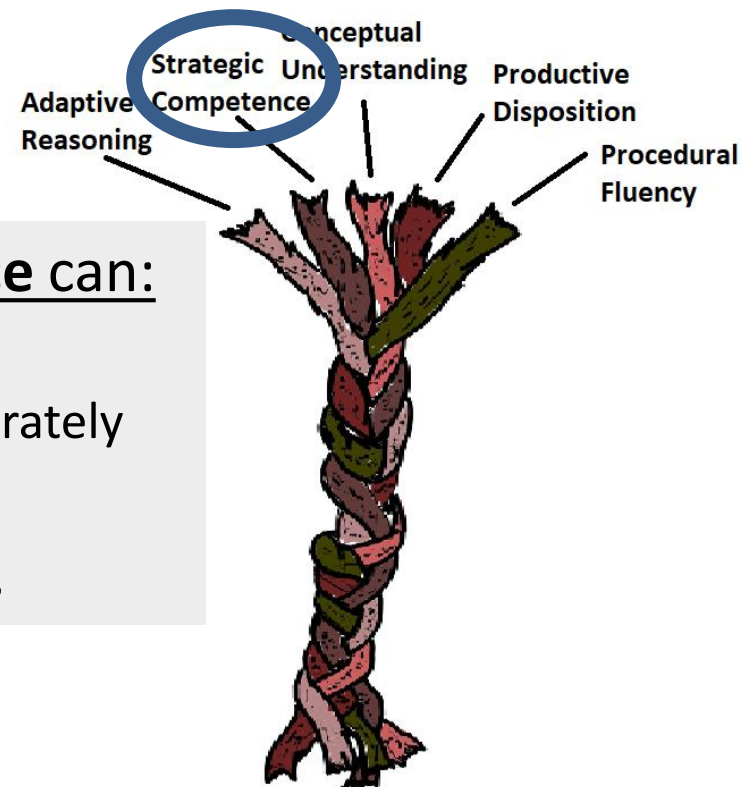
Adaptive Reasoning

Can you **prove** that in each of these images the area of the red quadrilateral is exactly half the area of the yellow square?



Strategic Competence

‘The ability to formulate, represent, and solve mathematical problems in both familiar and unfamiliar contexts.’



Students with **Strategic Competence** can:

- formulate mathematical problems
- represent mathematical problems accurately
- develop possible solutions
- evaluate the accuracy of their solutions

Strategic Competence

Question Posing

Is there something that you would like to research or investigate based on the image?

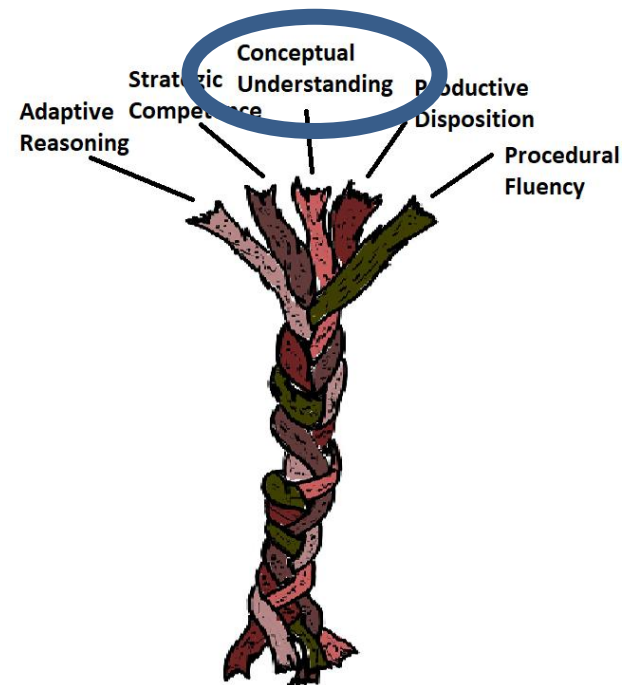


Conceptual Understanding

‘Comprehension of mathematical concepts, operations, and relations’

Students with Conceptual Understanding can:

- learn new ideas by connecting those ideas to what they already know
- represent mathematical situations in different ways
- see how the various representations connect with each other
- see the deeper similarities between superficially unrelated situations



Conceptual Understanding

<https://mathigon.org/>

Investigations

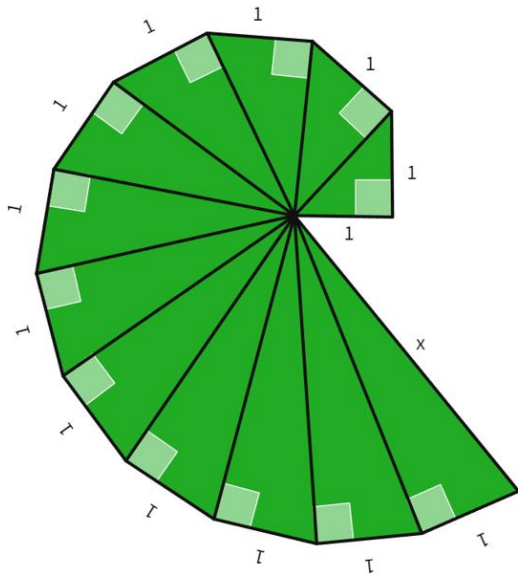
Right Angled Possibilities

If two of the sides of a right-angled triangle are 5cm and 6cm long, how many possibilities are there for the length of the third side?

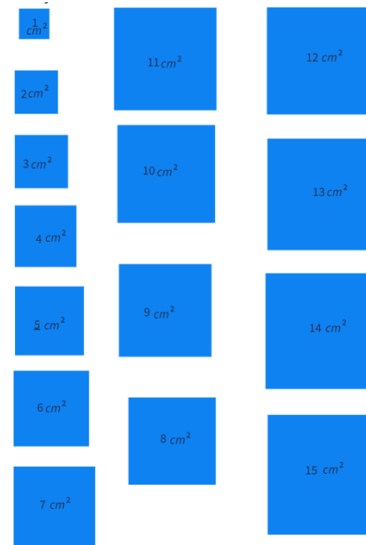
Can you explain your reasoning?

Pythagorean Shell

What missing side lengths can you find in this diagram?



Investigate what triangles can you form using side lengths from the squares (without overlapping squares)?



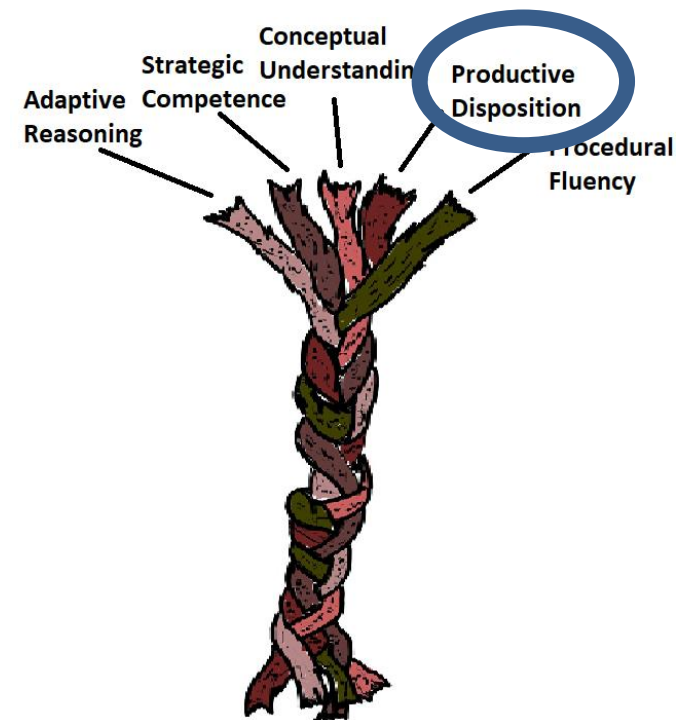
polypad.amplify.com/p/S3SGLq1yrr3E2A

Productive Disposition

‘Habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence, perseverance and one’s own efficacy’.

Students with **Productive Disposition** believe:

- mathematics should make sense
- that they can solve mathematical problems through productive struggle
- becoming mathematically proficient is worth the effort.



Productive Disposition



Real Life Connections

Brian Rushe
Dairy Farmer
Vice-President IFA

<https://tinyurl.com/3mb5xyuh>

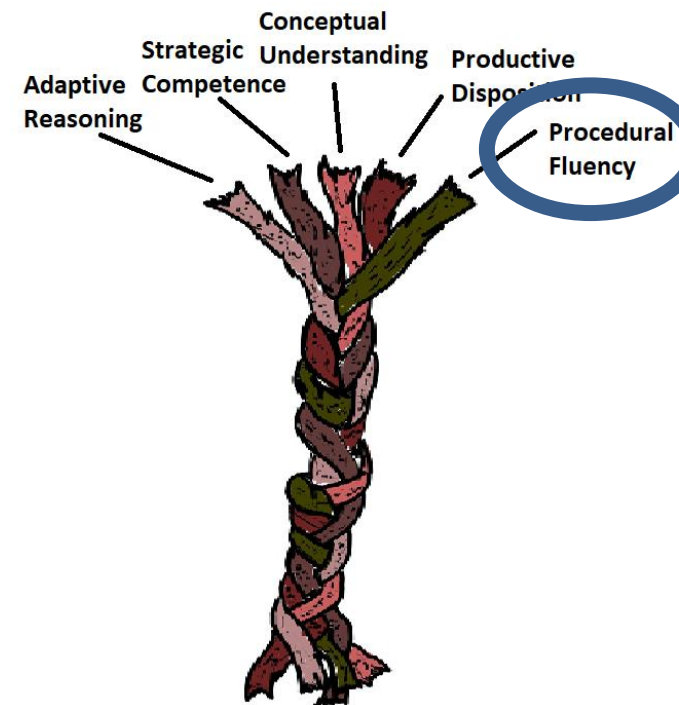


Procedural Fluency

Skill in carrying out procedures flexibly, accurately, efficiently, and appropriately

Students with **Procedural Fluency** can:

- estimate the result of a procedure
- practice of the skills they are learning
- apply procedures flexibly
- decide the appropriate tool for a given situation



Procedural Fluency

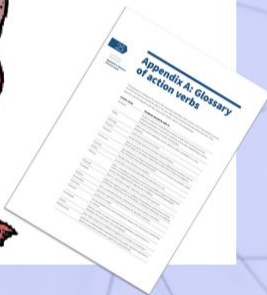
Practicing Procedures

Can you find every number between 1 and 20 using only 4's and any operation?



Components of Mathematical Proficiency

Conceptual Understanding
Strategic Competence
Adaptive Reasoning
Productive Disposition
Procedural Fluency



Adaptive Reasoning

Students with Adaptive Reasoning can:

- justify and explain ideas in order to make their reasoning clear

Productive Disposition

Students with Productive Disposition believe:

- mathematics should make sense
- that they can solve mathematical problems through productive struggle
- becoming mathematically proficient is worth the effort

Strategic Competence

Students with Strategic Competence can:

- formulate mathematical problems
- represent mathematical problems accurately
- develop possible solutions
- evaluate the accuracy of their solutions

Conceptual Understanding

Students with Conceptual Understanding can:

- learn new ideas by connecting those ideas to what they already know
- represent mathematical situations in different ways
- see how the various representations connect with each other
- see the deeper similarities between superficially unrelated situations



Procedural Fluency

Students with Procedural Fluency can:

- estimate the result of a procedure
- practice of the skills they are learning
- apply procedures flexibly
- decide the appropriate tool for a given situation

'...mathematical proficiency is conceptualised not as a one-dimensional trait but as having five interconnected and interwoven components.'

Junior Cycle Mathematics Specification p.5



Promising in theory...

... but not always easy in practice

“Students need to be regularly given high quality tasks that require them to engage with the processes promoted by the revised syllabuses”



Jeffes, J., Jones, E., Wilson, M., Lamont, E., Straw, S., Wheater, R., & Dawson, A. (2013). Research into the impact of Project Maths on student achievement, learning and motivation: final report. Slough: NFER

Mathematical Proficiency

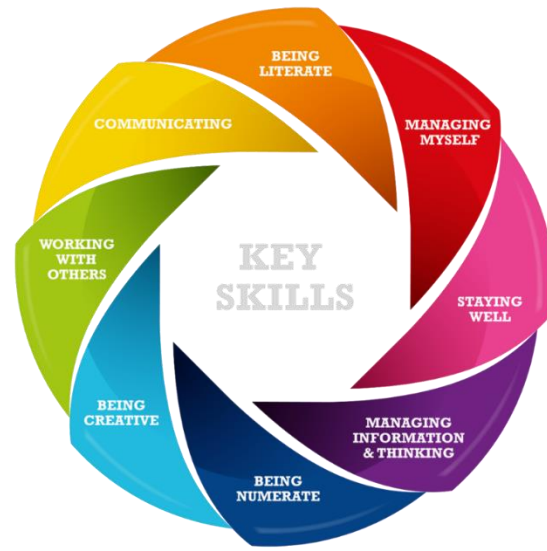
Is this the “goal” of maths education?

1. **conceptual understanding**—comprehension of mathematical concepts, operations, and relations
2. **procedural fluency**—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
3. **strategic competence**—ability to formulate, represent, and solve mathematical problems in both familiar and unfamiliar contexts
4. **adaptive reasoning**—capacity for logical thought, reflection, explanation, justification and communication
5. **productive disposition**—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence, perseverance and one’s own efficacy

(Kilpatrick et al., 2001, p117)

Junior Cycle Maths Specification

<https://www.jct.ie/perch/resources/maths/junior-cycle-mathematics-specification-2018.pdf>



Classroom-based Assessments (CBAs):

Relate to the priorities for learning and teaching in mathematics, with a particular emphasis on problem solving and communicating.

(Specification for Junior Cycle Mathematics, p22)



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Pedagogical Framework

1. Realistic Mathematics Education (RME)
2. Student-centred, innovative pedagogies – Bridge21

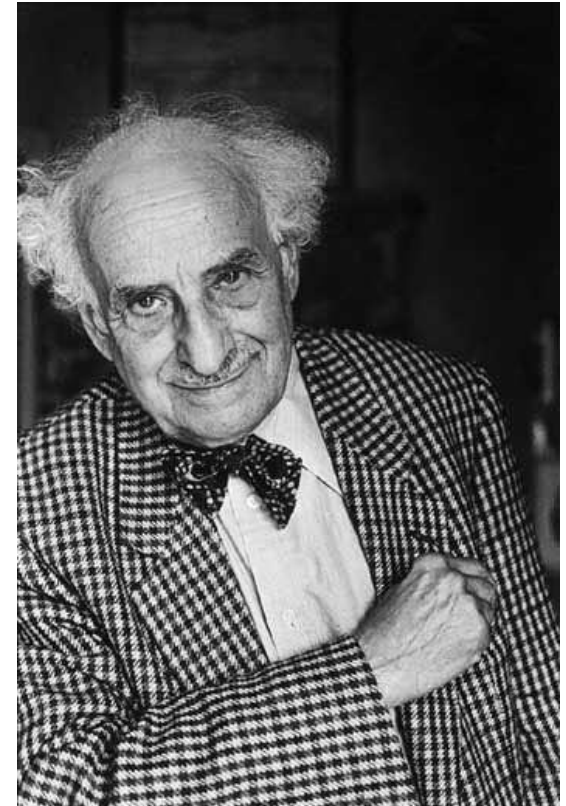
Realistic Mathematics Education

RME

“RME is an approach to mathematics education that involves students developing their understanding by exploring and solving problems set in contexts that engage their interest”

(Tangney, Bray & Oldham, 2015)

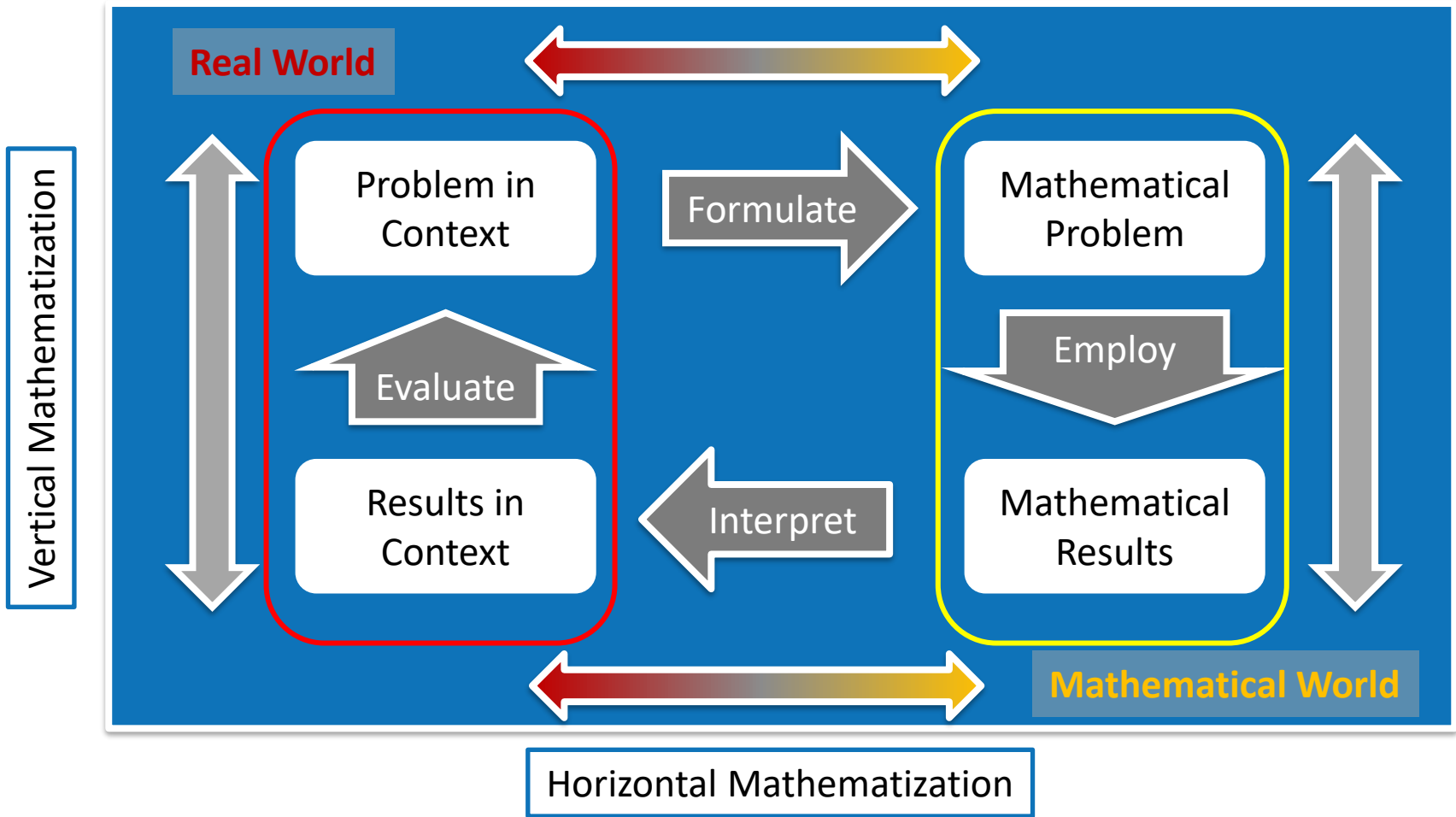
1. The importance of problems set in **contexts** that are real to the students.
2. The attention paid to the development of **models**
3. The contributions of the students by means of their own **productions** and **constructions**.
4. The **interactive** character of the learning process.
5. The **intertwinement** of learning strands.



Hans Freudenthal (1905 - 1990)

Realistic Mathematics Education

RME: Mathematization



Compare this to the NCCA documentation

Guidelines for the Classroom-Based Assessments

Classroom-based Assessment 1: Mathematical Investigation

The Mathematical Investigation comprises of four areas of activity:

1. defining the problem statement,
2. finding a strategy and translating it to mathematics (if necessary),
3. engaging with the problem and solving it if possible,
4. and interpreting the solution in the context of the original problem.

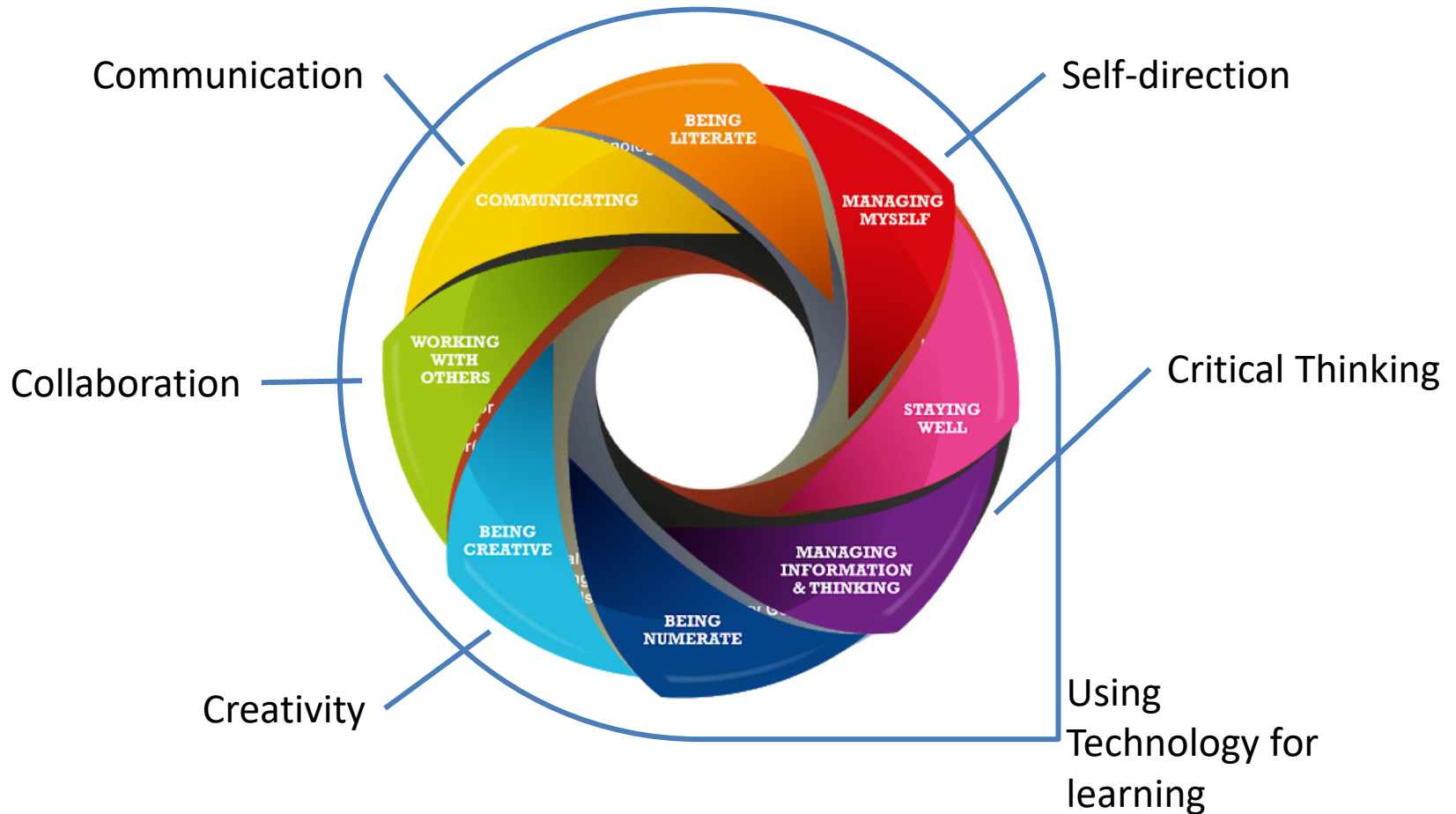


Students may ***collaborate with peers*** at various stages during the process and then compile a report of their findings individually.

National Council of Curriculum and Assessment. 2019. Guidelines for the classroom-based assessments and assessment task. Dublin: NCCA, p12



Bridge21 Pedagogical Model



The Classroom Contract

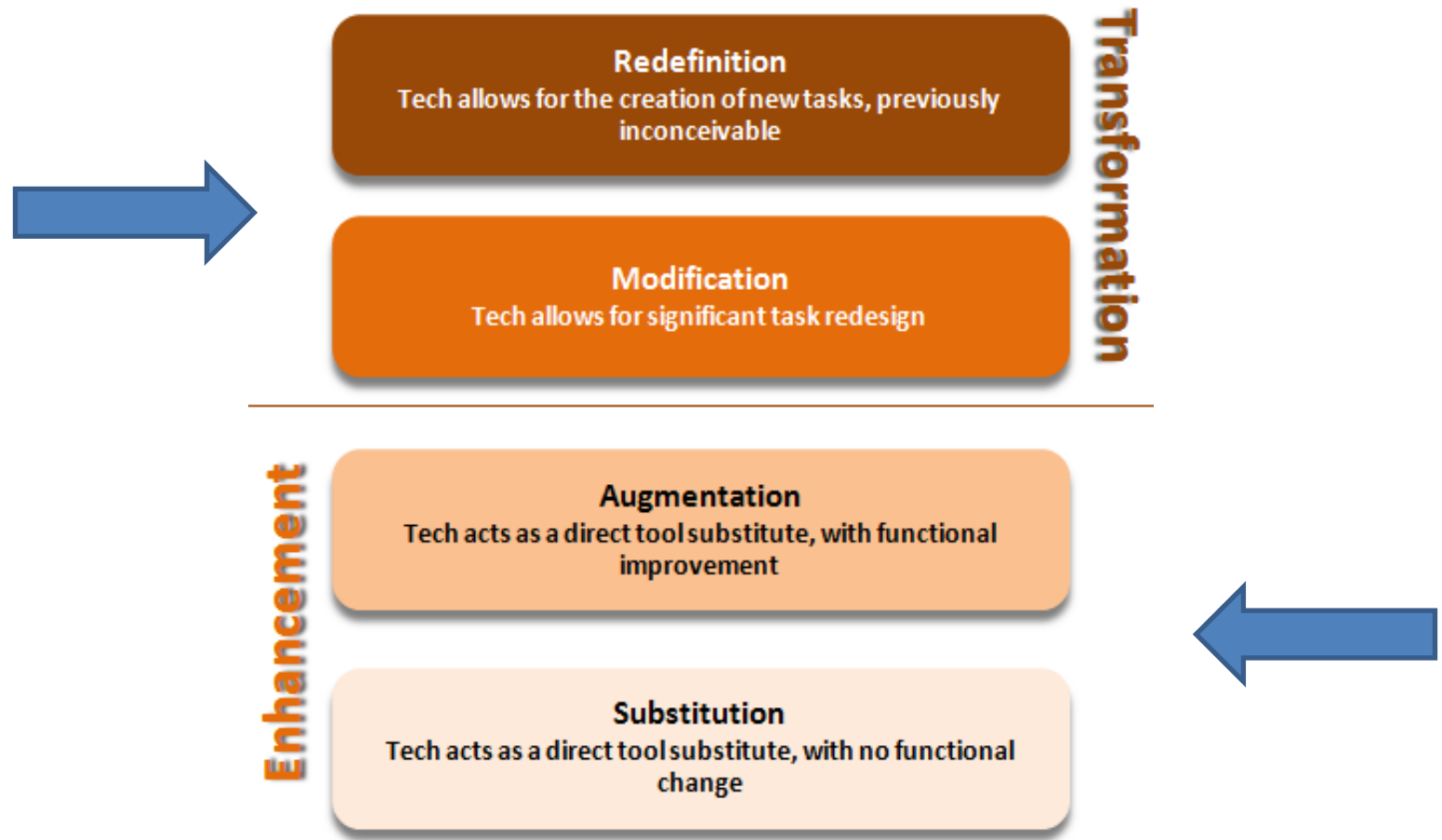


Students take more responsibility for their own and their peers' learning.

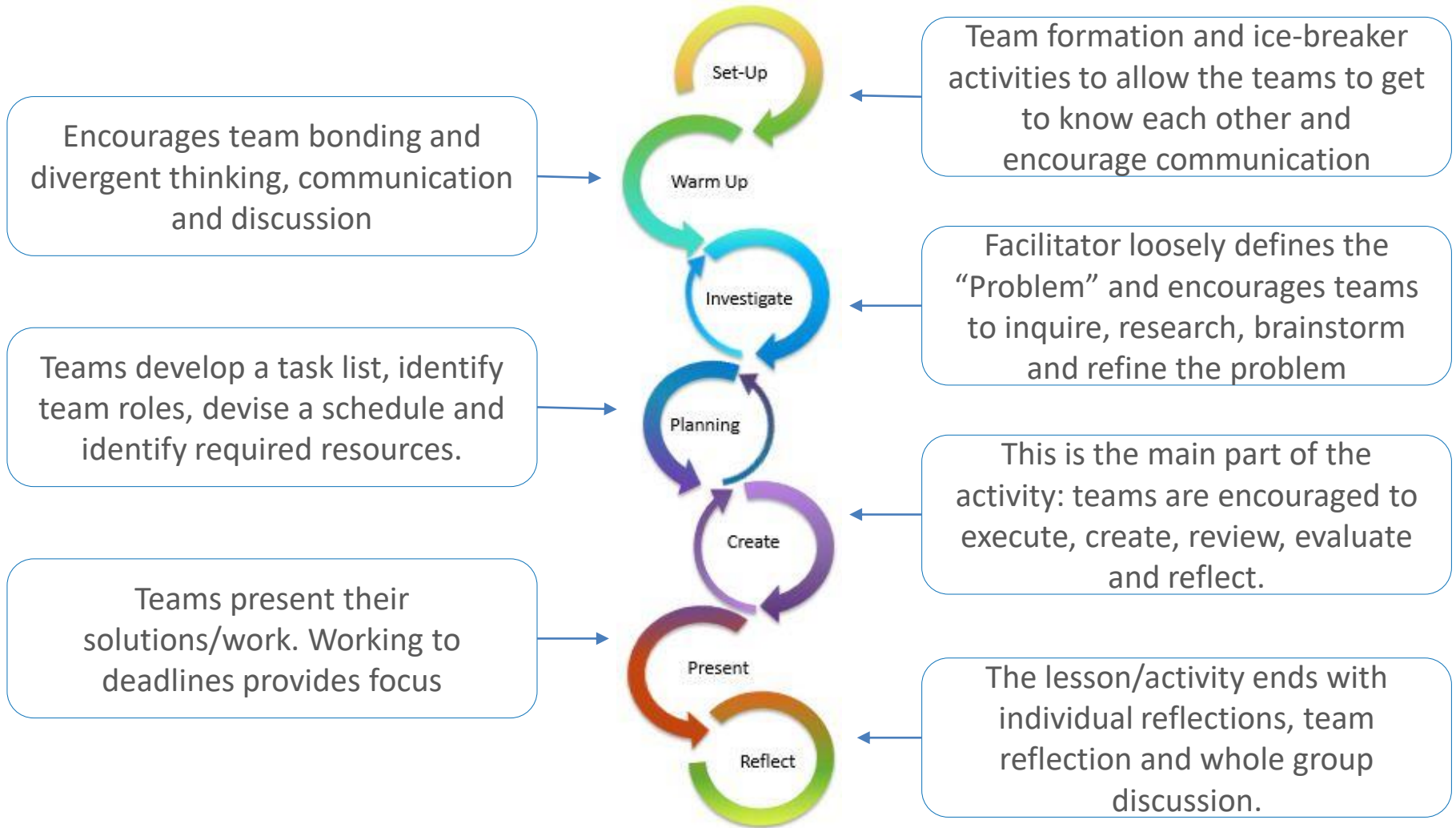
Bridge21 Groupwork Model



Levels of Technology Adoption

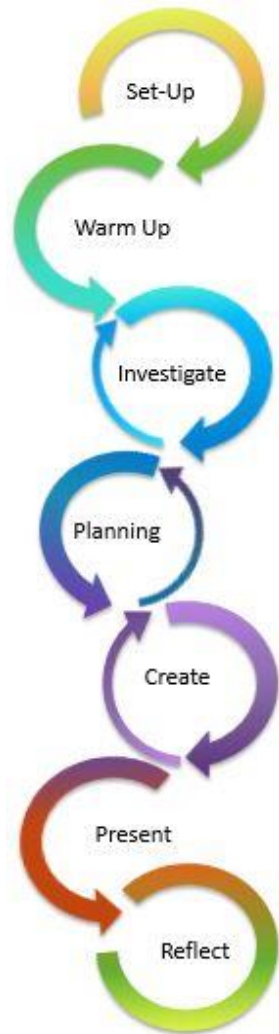


Bridge21 Activity Model



Bridge21 Activity Model

Mathematics



- **Set-Up:** Ice breaker and team formation
- **Warm Up:** Divergent thinking activity
- **Investigate:** Explanation of the problem context.
- **Plan:** Group planning.
- **Create:** Exploration with resources.
 - In the field.
 - In the classroom.
- **Create:** Modelling and Calculation:
 - Analysis and Synthesis.
- **Present:** Competition and/or Presentations.
- **Reflect:** Reflection and Discussion.



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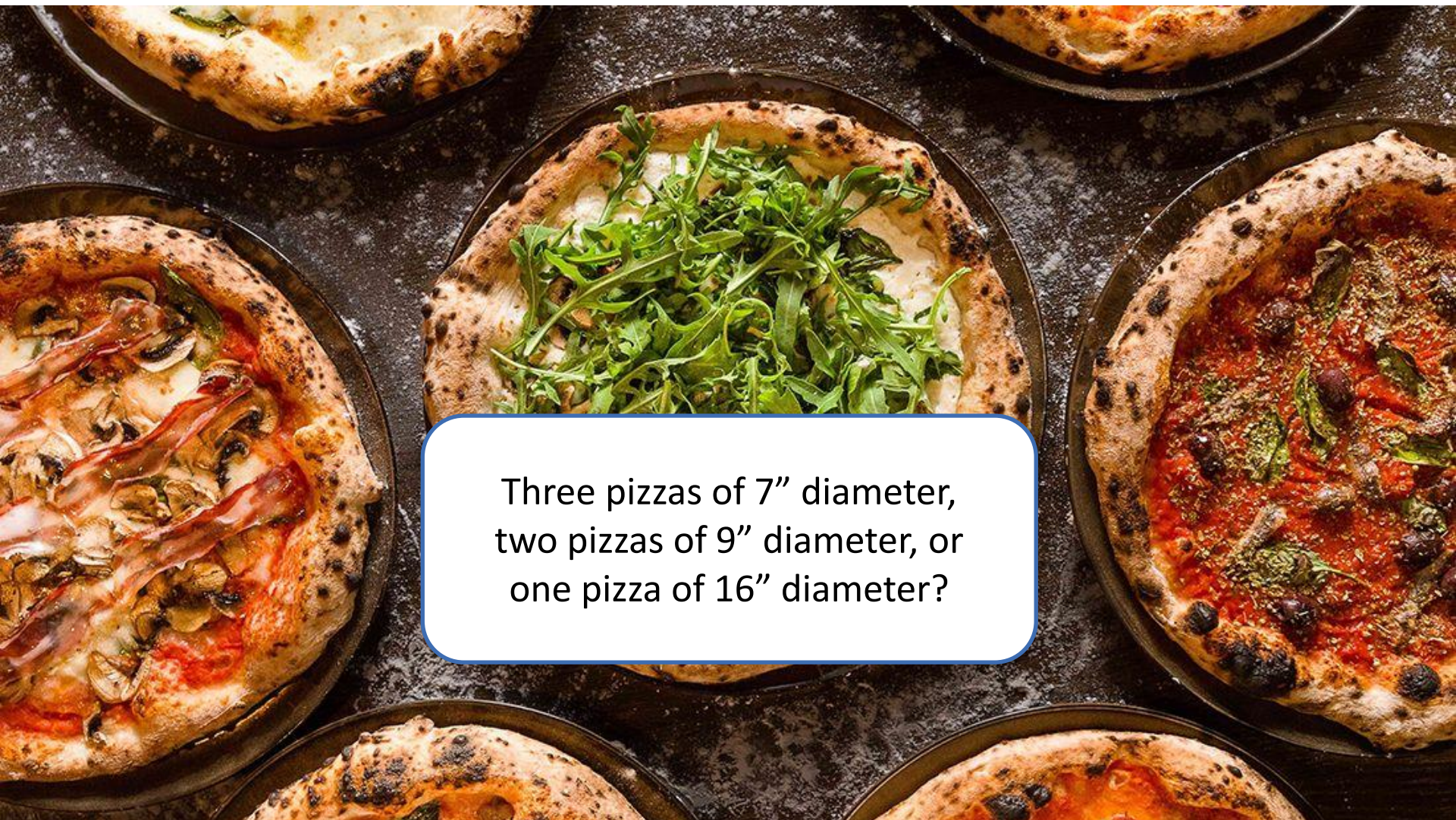
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Activity 1



Which is bigger...?

Think fast...



Three pizzas of 7" diameter,
two pizzas of 9" diameter, or
one pizza of 16" diameter?

Pizza Maths

Challenge – the Big Problem:

You have a budget of €50 to feed your team

1. How can you get the most Margherita pizza?
2. What area of pizza is that?

Possible Websites:

- <https://www.just-eat.ie/restaurants-teach-pizza-dublin/menu>
- <https://www.just-eat.ie/restaurants-papa-johns-westmoreland-street-dublin2/menu>
- <https://www.just-eat.ie/restaurants-basewoodfiredpizza-glenageary/menu>





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15 minute break!





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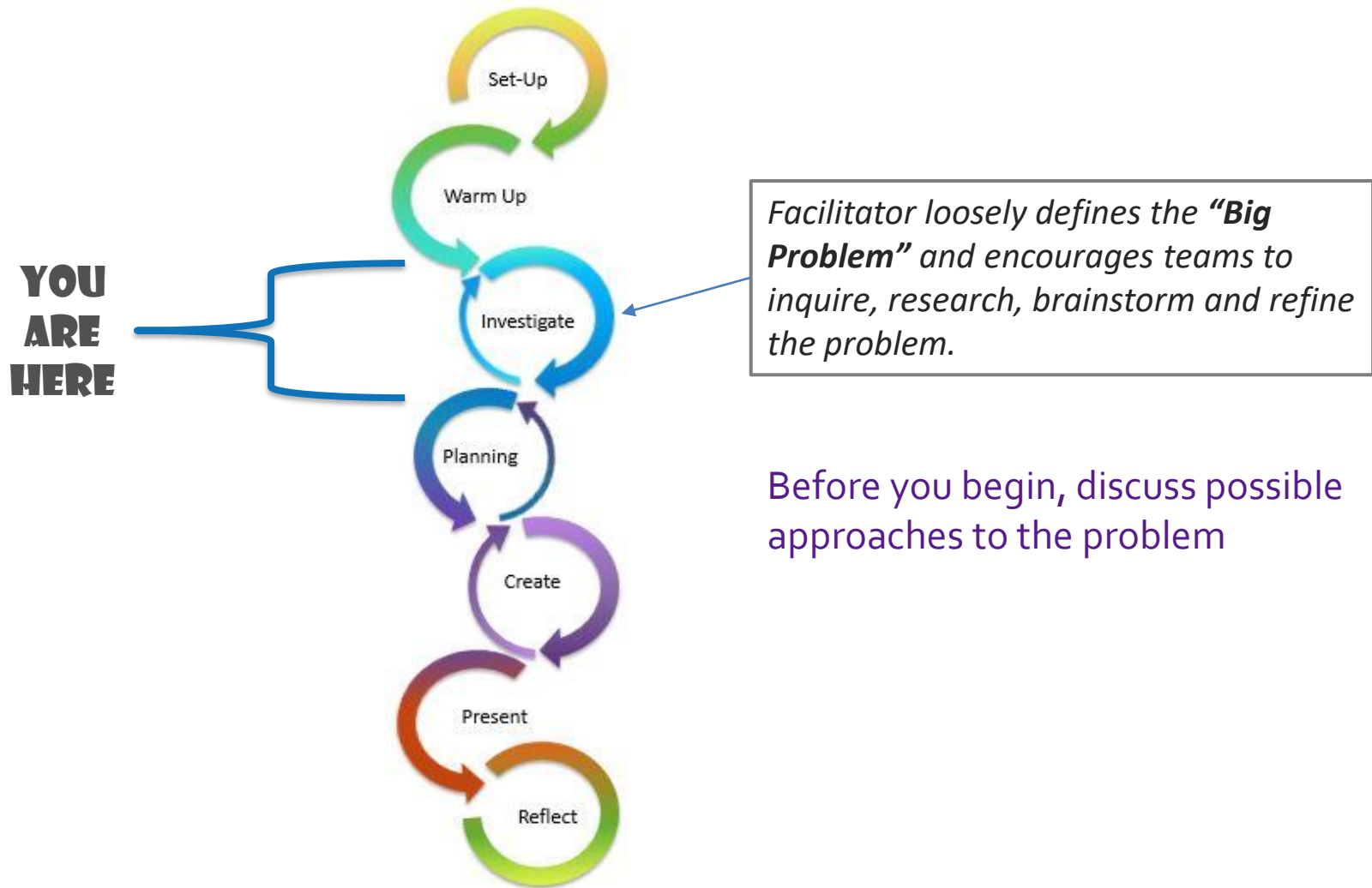
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Investigate Phase



Bridge21 Activity Model

Investigate



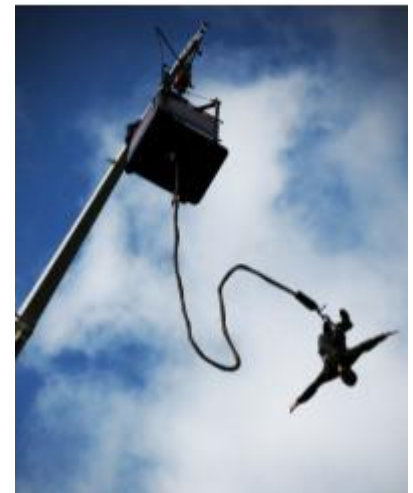
The Barbie Bungee

“Big Problem” or Challenge:

Using a doll, rubber bands, and some free software, calculate how many bands it would take to give Barbie an exhilarating, but safe jump from a height?



Spreadsheet



<http://illuminations.nctm.org/Lesson.aspx?id=2157>

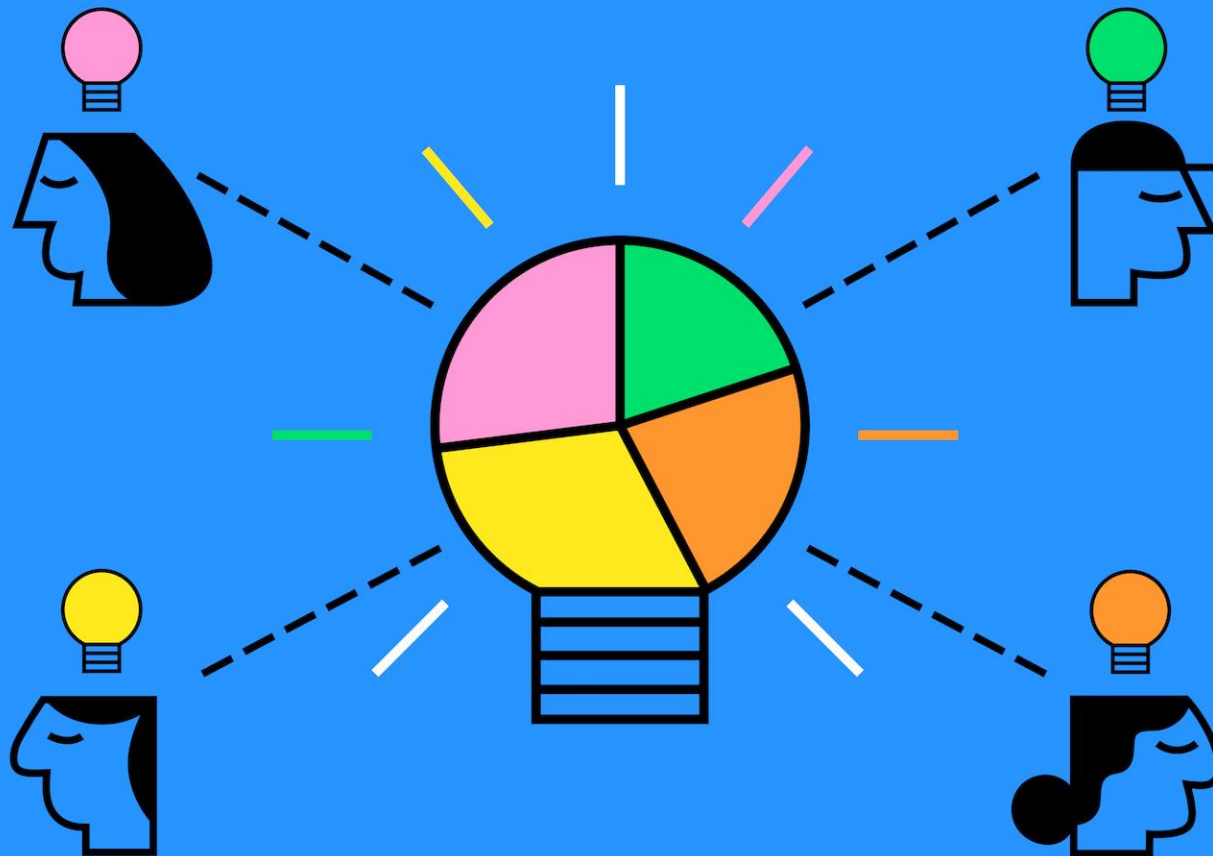
Homemade BeanBag

- Sock/plastic bag filled with
 - Rice
 - Lentils
 - Baking beans
 - Stones
 - Sugar
 - ...

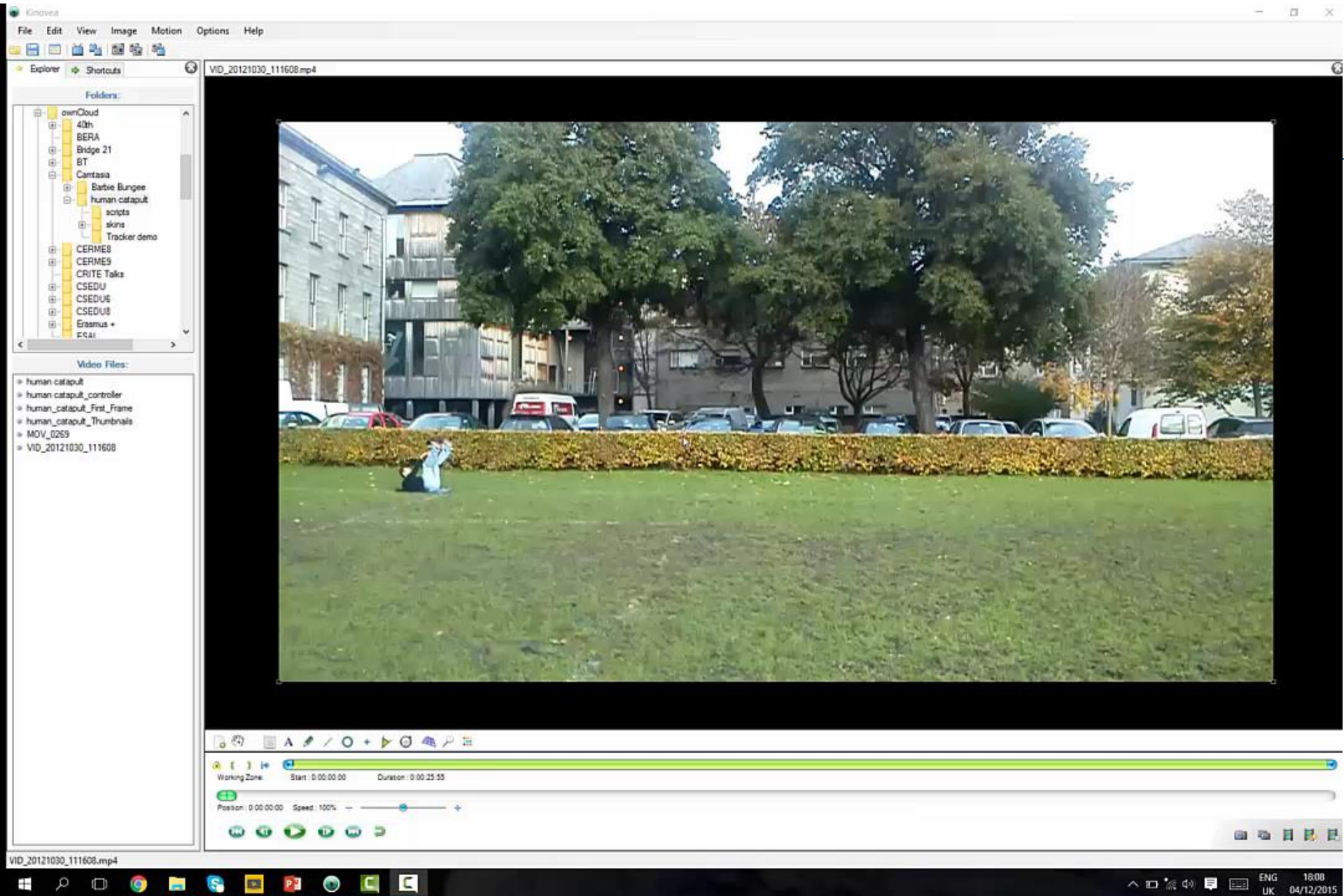


Team Task 1

Brainstorm ideas



Kinovea Demo





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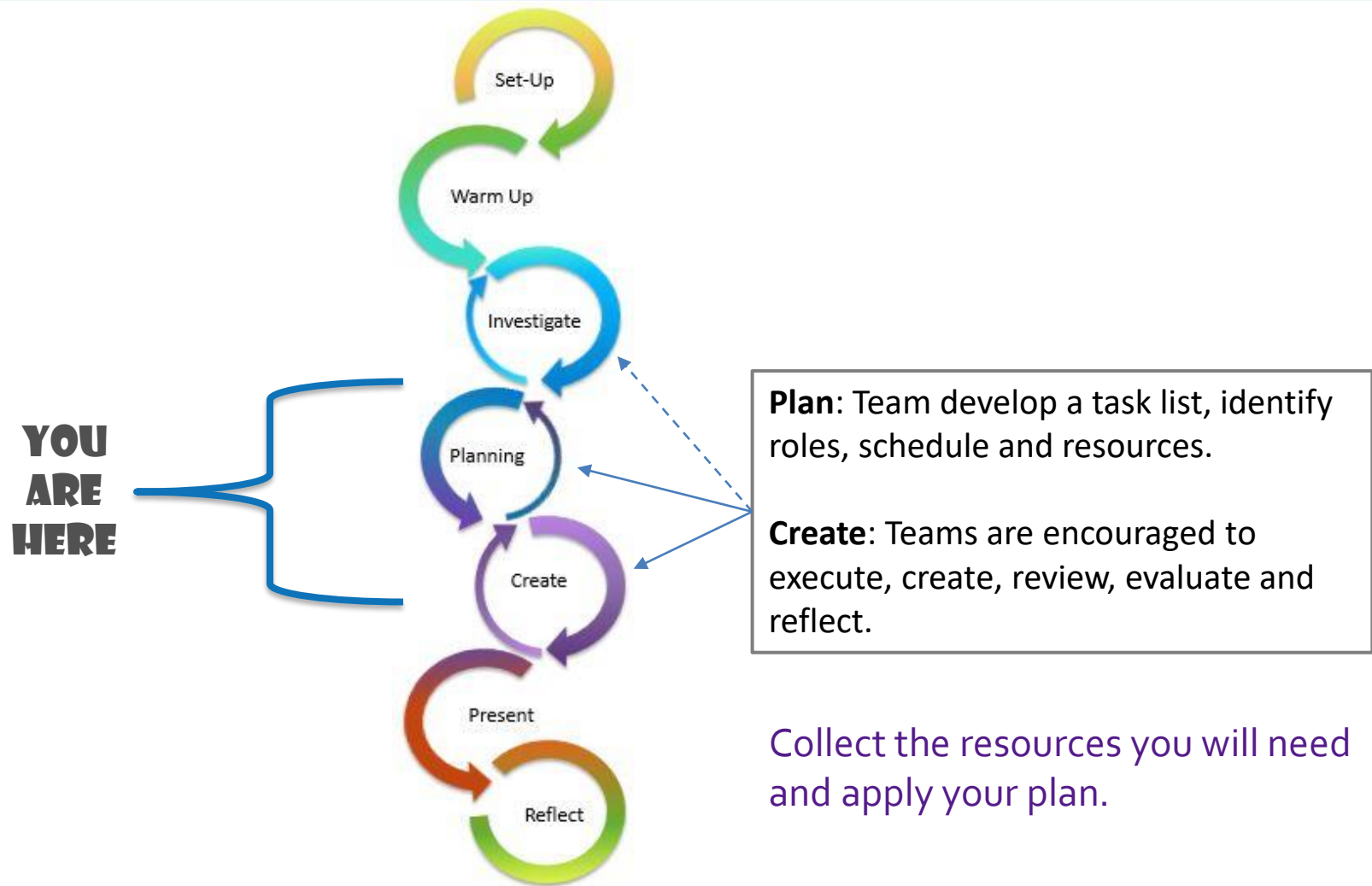
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Plan – Create – Investigate Cycle

Bridge21 Activity Model

Plan – Create – Reflect - Investigate



Extra Scaffolding

<https://tinyurl.com/ExpressDiving>





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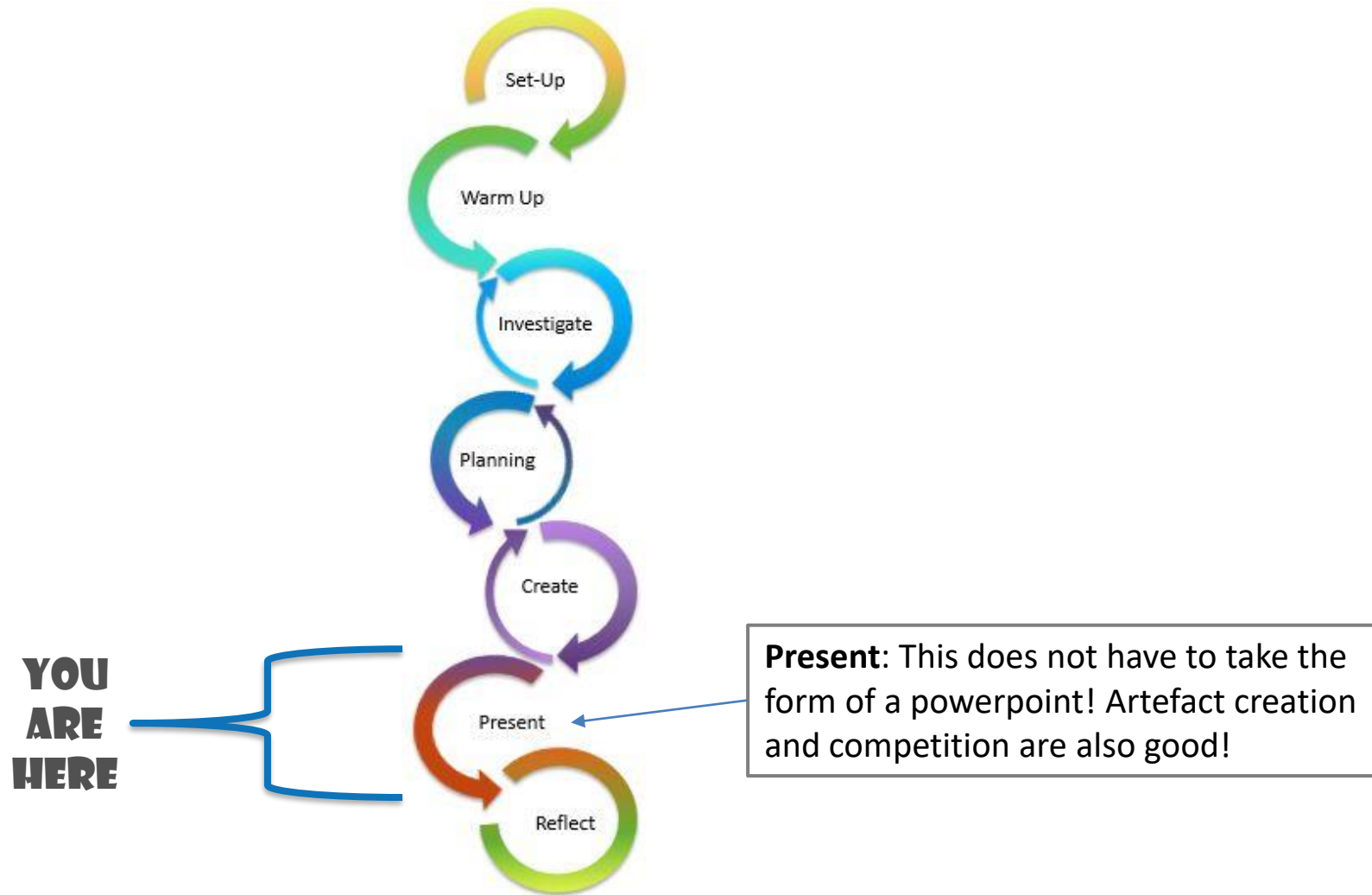
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Present your work...



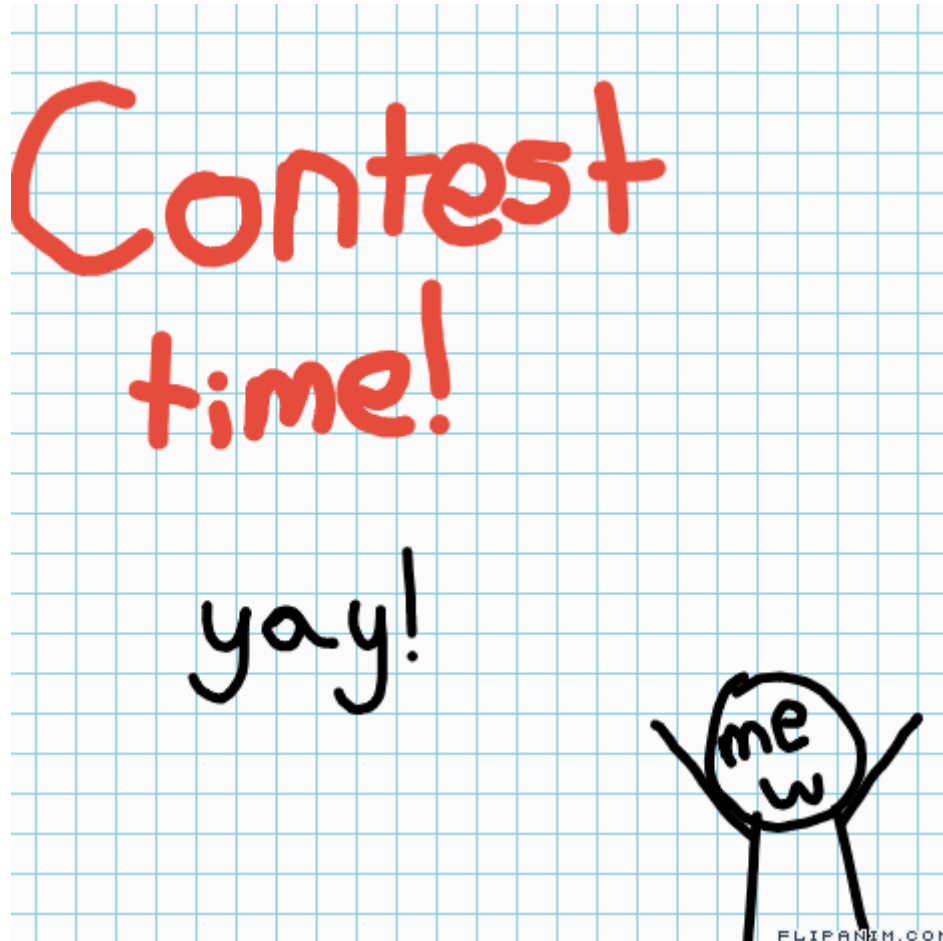
Bridge21 Activity Model

Plan – Create – Reflect - Investigate



Competition time!

Show us what you got!





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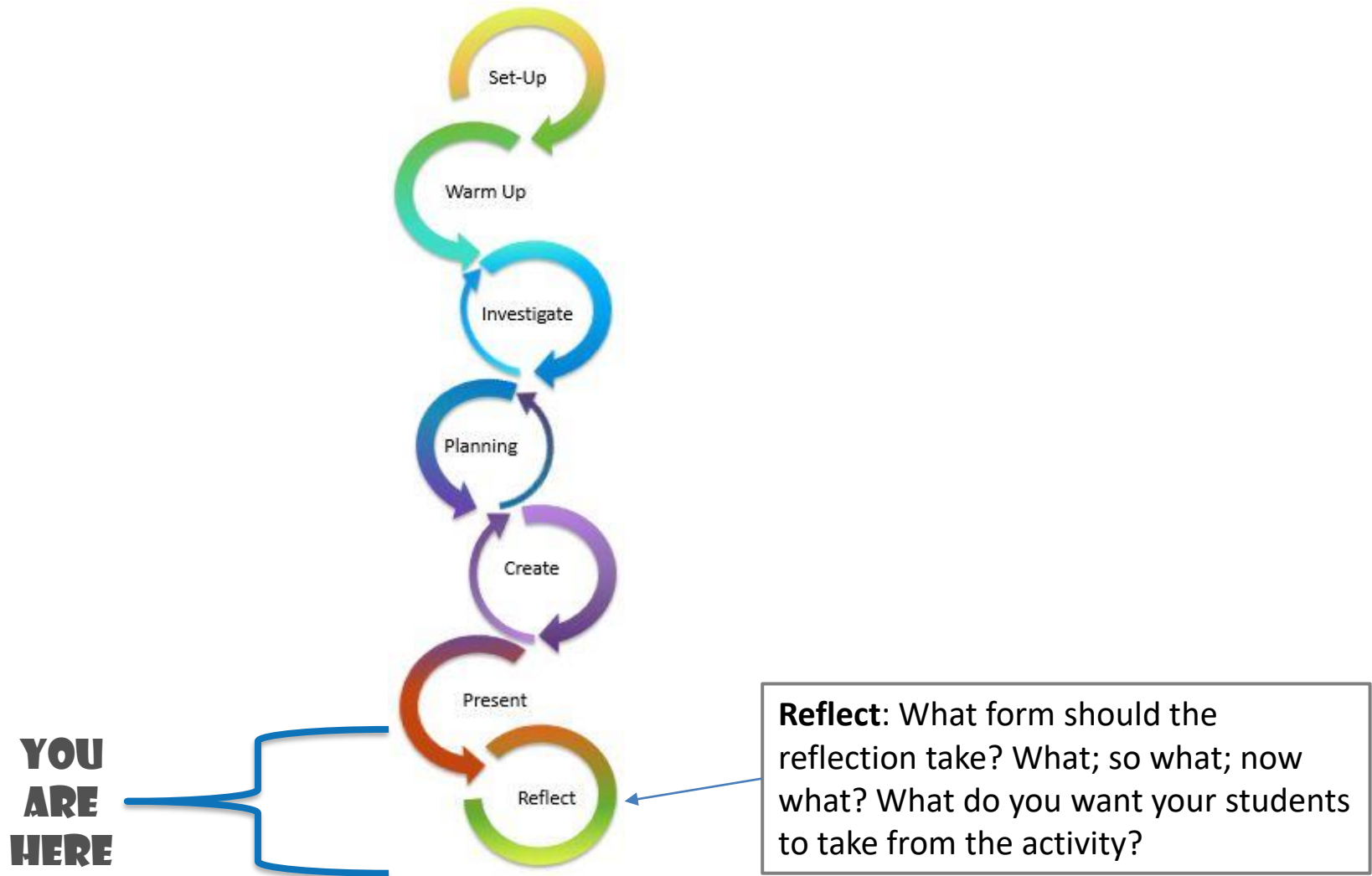
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Reflect

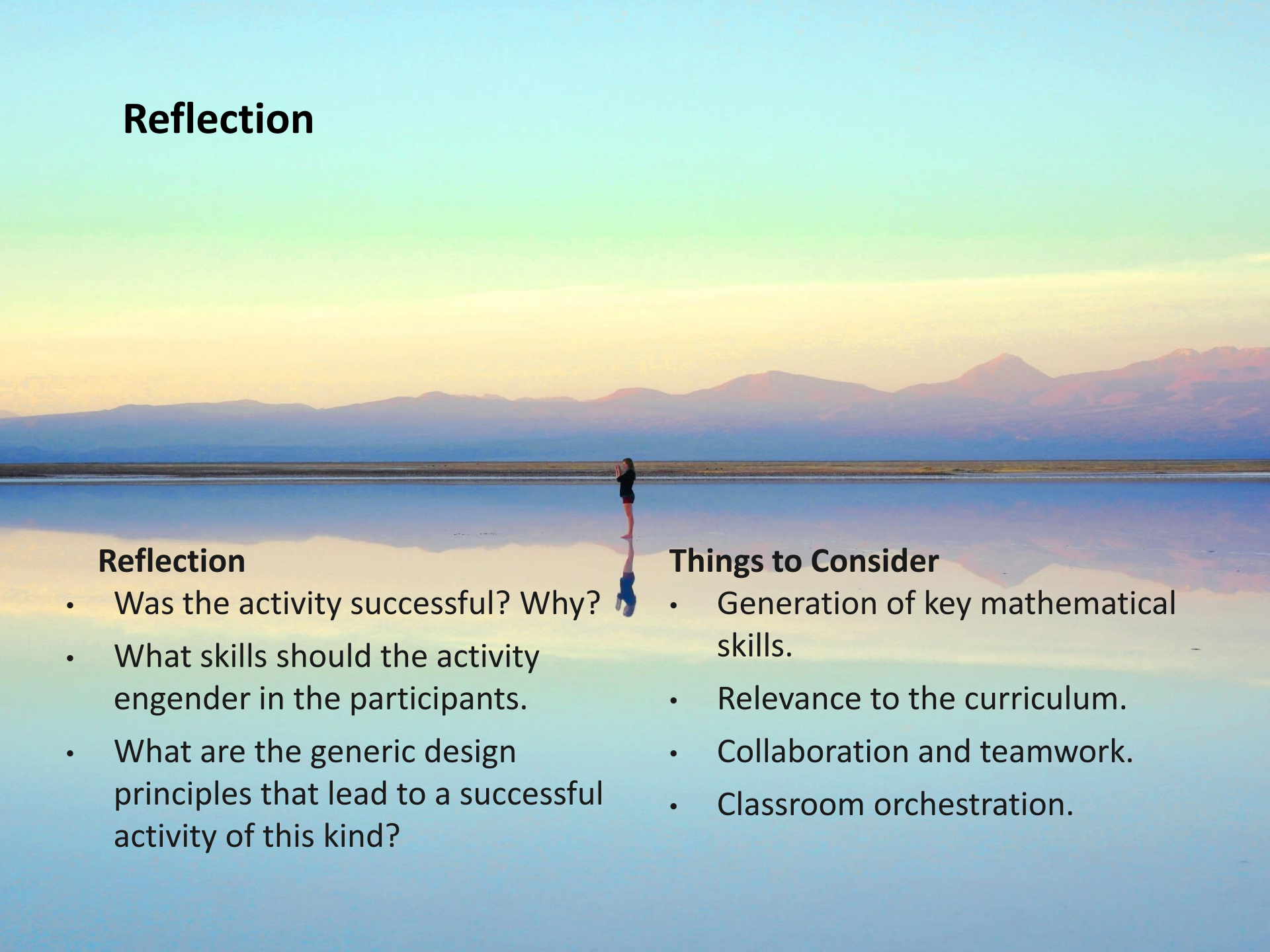


Bridge21 Activity Model

Plan – Create – Reflect - Investigate



Reflection



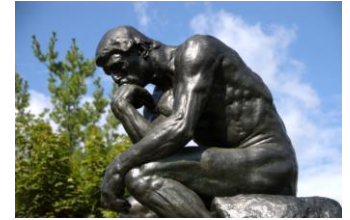
Reflection

- Was the activity successful? Why?
- What skills should the activity engender in the participants.
- What are the generic design principles that lead to a successful activity of this kind?

Things to Consider

- Generation of key mathematical skills.
- Relevance to the curriculum.
- Collaboration and teamwork.
- Classroom orchestration.

Reflection



Successful because...

Need to address...

That's all folks!





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Thank You

